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TRAJECTORY METHODS IN MISSION ANALYSIS FOR LOW-THRUST VEHICLES

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INTRODUCTION

Once an interplanetary mission is defined, such as the delivery of a scientific payload to a planet or the execution of an exploratory manned round trip to Mars, a preliminary planning phase must be entered that may be defined as mission analysis. The ultimate, but never completely attained, goal of mission analysis is the accurate determination of such things as cost, feasibility, and design requirements of proposed vehicle systems.

To make the problem more definitive, a figure of merit is usually chosen and is optimized under the influence of a host of practical and analytical constraints. One of the most readily accepted figures of merit that may be chosen to be optimized in mission analysis is the vehicle gross weight for some specified payload. Interplanetary missions with low-thrust rocket vehicles are attractive because they promise low propellant weight requirements. In the case of electric propulsion, this effect is primarily offset by high powerplant weight requirements and thrustor capability limitations. But it is also desirous to examine the effect of performance features and limitations of many other vehicle subsystems, such as propellant storage and feed systems and biological radiation shielding. Therefore, total mission analysis involves appropriate compromise of all parts of the integrated vehicle with mission profile and energy requirements.

This paper is concerned with trajectory analysis, a field of study that is important in mission analysis. The purpose of this paper is to present some of the standard procedures and results of low-thrust trajectory methods that are in use by the Mission Analysis Branch at the NASA Lewis Research Center and to describe a more recently developed trajectory analysis technique that has proved to be quite useful in mission analysis.

Low-Thrust Trajectory Considerations

Trajectory energy requirements dictate not only the amount of propellant needed but also the basic capability of a vehicle to perform a given mission profile.

In high-thrust trajectory analysis, energy requirements are evaluated independent of vehicle characteristics, using the familiar "ΔV." But in low-thrust trajectories, the energy requirements are affected by the thrust and jet velocity. Conversely, the required performance of vehicle subsystems, such as optimum thrustor jet velocity and thrust, are strongly affected by trajectory energy requirements. Therefore, the trajectory calculations must usually be made an integral part of the mission analysis.

When the total mission optimization is attempted, much interplay occurs between the trajectory itself and the trajectory-dependent vehicle subsystems, thereby continuously sharpening the definition of the desired trajectory and vehicle design. A complete mission analysis entails the calculation of many trajectories for low-thrust vehicles in the search for optimum vehicle design parameters. The trajectory calculations must recognize certain practical constraints that apply to the vehicle.

It is usually assumed that the vehicle employs some type of electric propulsion. For maximum performance during propulsion phases, the vehicle should operate at the maximum available power level. Therefore, an immediate operating constraint that can be placed on the vehicle is that it uses a prespecified jet power output capability. For example, trajectory solutions have been made for a hypothetical electric rocket with a variable thrust and jet velocity capability but constrained by constant jet power.[1,2] This "variable-thrust" trajectory technique leads to the best possible rocket performance at any given jet power.

However, the constant thrust thrustor is probably more representative of early vehicles. Therefore, the particular low-thrust trajectories concentrated upon in this paper assume that thrustors are constrained to operate not only at constant jet power but also at constant thrust and jet velocity with on-off propulsion periods.[3] In other words, the thrustor simply operates at only one rating, or not at all. This mode of operation is referred to herein as "power-coast-power." Power-coast-power trajectories have proved to be far more difficult to analyze than variable-thrust trajectories.

The requirement that the trajectory actually represents a transfer between two specified positions and velocities and the desire that the transfer be as efficient as possible, act as further constraints in the problem. These constraints necessitate the solution of the "two-point boundary value problem" and the "minimum-propellant trajectory."

Trajectory problems become two-point boundary value problems when some initial conditions of a solution must be found to satisfy prespecified final conditions. Low-thrust trajectories are characterized by very long propulsion periods during which the thrust vector must be controlled according to some stipulated program. Solution of the proper equations of motion calls for, except in the most simple cases, numerical integration methods. The two-point boundary value problem must then be solved as an initial-value problem, where a complete set of values for all the variables being integrated must be known at the initial point. The correct initial set is only partly known. Iterative guessing techniques must be employed until the complete set of initial-point variables and the required thrust-control program necessary to meet the speci-

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fied end conditions are found. Attempts at alleviating the two-point boundary value problem for low thrust can sometimes be made by arbitrary policies for applying the available acceleration vector over the propulsion period.[4,5] Nonoptimum thrust vector control, however, can result in excessive propellant requirements.

Each time performance characteristics of the vehicle are stipulated, along with any other constraints that might be included, the most efficient thrust vector control program should be found. This is a part of the problem that is always present in case of low-thrust propulsion. Therefore, when the term minimum-propellant trajectory is used, it simply means that the most efficient application of the available thrust has been used in the trajectory solution to minimize energy requirements. Regardless of the number and type of constraints placed on a trajectory and vehicle, optimum overall missions will be fundamentally constructed with minimum propellant trajectories.

When it is specified that the resultant trajectory solution must have the most efficient thrust vectoring program in order that some measure of vehicle performance is optimal (e.g., minimum propellant consumption), optimization techniques such as the variational calculus are needed. This is an extension of the low-thrust trajectory problem that has received much attention. The two-point boundary value problem must still be solved with the variational calculus, but vehicle performance is simultaneously optimized, giving more significance to the trajectory solution. For the power-coast-power problem, the thrust and jet velocity are held constant at prescribed values, while the variational solution determines the optimum thrust direction and the placement of a coast period to result in minimum propellant consumption.

Typical Trajectory Characteristics

Figure 1 is an example of a "map" of variational power-coast-power trajectory solutions. This figure illustrates the characteristics of one-way Earth to Mars heliocentric transfers at one initial thrust to weight ratio and specific impulse. Circular, coplanar heliocentric orbits have been assumed for Earth and Mars. Travel angle (heliocentric central angle) is plotted against travel time along contours of constant final mass ratio M_f/M_o. Constant final mass ratio is synonomous with constant propellant consumption and is a measure of the energy requirement of the trajectories. Each travel angle and time pair constitute a solution to a two-point boundary value problem with the calculus of variations. For the illustrated thrust and specific impulse, many time and angle combinations exist within the boundary marked "all-propulsion." However, no solutions exist outside the boundary. All-propulsion, or zero coast time, solutions result when the energy requirements are so severe that the thrustor must operate continuously over the available time.

Any one point on a map such as figure 1 is an optimum solution, in the sense of minimum propellant consumed for the stated thrust, specific impulse, and other given operating constraints. If proper compromises between trajectory energy requirements and vehicle performance levels are to be made, one map such as this is of limited value to mission analysis. Each new value of thrust to weight ratio or specific impulse raises the need for yet another map. Even at a fixed one-way time and angle combination, variational solutions possess different final mass ratios (and, therefore, energy requirements) for each new thrust to weight ratio or specific impulse. An example of the effect of thrust alone is given in figure 2.

In figure 2, a curve of final mass ratio versus initial acceleration (due to thrust) is shown. This curve is for an arbitrary, fixed travel angle and time combination with specific impulse fixed at 6000 seconds.

The lowest final mass ratio and initial acceleration recorded on the curve is an all-propulsion solution. Note that at the low-acceleration end of the curve, mass ratio can be seen to vary quite strongly. This sensitivity decreased markedly as acceleration is increased.

Optimum Probe and Round-Trip Missions

With the goal of low-thrust mission analysis in mind, computer programs have been developed to study orbiting probe and round-trip missions, using the calculus of variations for power-coast-power heliocentric trajectories. Examples of these calculations are discussed here. The performance chart of a typical orbiting probe trip to Mars is shown in figure 3 where final mass ratio is presented as a function of initial acceleration for a range of jet power to mass ratio in watts per kilogram.

The problem model assumes a two-dimensional solar system with the planets in circular orbits about the Sun. The mission trajectory is actually a "patched" sequence of two-body planetocentric and heliocentric phases. The heliocentric variational trajectories begin and end with the vehicle in circular orbit about the Sun at each respective planet-Sun radius. Low-thrust planetocentric escape and capture phases are included in this mission. The mission commences with the probe vehicle in low orbit about the Earth and ceases with it in low orbit about Mars, hence, the term "orbiting probe." Much simplification is introduced by the assumption that near-planet escape and capture maneuvers are two-body trajectories (spirals) influenced only by the planet in question. It is assumed that the planet ceases to exert influence when the vehicle is at escape energy, because, for low thrust, escape occurs at large radii relative to the planets. Actual calculations of the sprials could be lengthy. Much time is saved by using precalculated generalized spiral solutions[4] to which empirical curves have been fitted. In this way, the appropriate time and propellant consumption of each spiral maneuver may be charged to the mission.

Trajectory calculations for optimum orbiting probe missions make further use of the calculus of variations to ensure that the most optimum heliocentric travel angle is used for any stipulated travel time. In this way, travel angle is not an independent parameter for optimum probe missions. The particular map shown in figure 3 is for a specified total time of 300 days from low Earth orbit to a low Mars orbit.

The boundary curve at the left side of the chart consists of all-propulsion solutions. This boundary represents the lowest possible acceleration that may be used at each jet power to mass ratio.

In figure 4, the same problem model and technique has been used to generate performance maps for optimum round trips to Mars. Each chart of this type is for a given mission time (from low Earth orbit back to low Earth orbit) and wait time (in low Mars orbit), in this case 380 and 10 days, respectively. Again, low-thrust planetocentric spirals are included in the solutions. The round trip final mass ratio M_f/M_o is shown as a function of initial acceleration along lines of constant jet power to mass.

There is a major difference between trajectory calculations for optimum round trips and for optimum probes that is not obvious in figure 4. In the round-trip problem, the calculus of variations is further applied to result in the optimum pair of outbound and return variational trajectories in such a way that propellant requirements for the complete trip be minimized.[6] Therefore, heliocentric travel angles of the outbound and return trajectories do not correspond to optimum probe cases. Each point on the round-trip map represents the best possible combination of travel time and angle for outbound and return minimum-propellant trajectories for the given problem model, mission profile, and operating constraints.

As with the orbiting probe, a boundary curve appears as a characteristic of these charts. For the case shown here, an all-propulsion trajectory on the outbound leg of the round-trip mission establishes a lower limit of possible acceleration to accomplish the trip at each power to mass ratio.

Jet power to mass ratio is used as a field parameter on both round-trip and probe maps rather than specific impulse; however, this is a completely arbitrary choice. At each power to mass and thrust to mass, a specific impulse (jet velocity) is defined, since jet power is directly proportional to thrust and jet velocity.

Terminal calculations of necessary vehicle component weights can be applied to basic mission performance charts, such as figures 3 and 4, to result in nearly exact relations between vehicle gross weight and useful payload. A sophisticated view of those components may be taken, such as ion thrustors with an efficiency related to specific impulse, engine weight related to thrust, and, of course, electric powerplant weight related to power.

It would be most desirable to base all low-thrust mission studies on accurate variational trajectories; however, experience has shown that ambitious mission studies with variational power-coast-power trajectories involve lengthy and, therefore, costly computer calculation. Although they do add to the complexity of the problem, variational trajectory methods are not the chief source of difficulty. The major problem area is the need for repeated solutions of the ever-present two-point boundary value problem with numerically integrated trajectories. Whatever the major source of difficulty, preliminary design studies for low-thrust missions are too often hindered by the complexity of the trajectory calculations. The effects of changes in mission profile, such as different parking orbit radii, mission and wait times, and supercircular aerodynamic reentry options, become very difficult to evaluate.

If guide lines for the feasibility and design requirements of low-thrust vehicles are to be drawn, trajectory calculations must be available with higher speed and flexibility. Faster and simpler trajectory solution approaches, even with the admission of some degree of error, would allow a wide-range analysis of many important areas in the overall mission problem.

Approximation by Correlation

Approximate trajectory solutions can play an important role in the low-thrust mission problem if such solutions can satisfy the speed and flexibility requirements already mentioned. Some degree of approximation is always involved in any calculation, as evidenced in the discussion of the probe and round-trip-problem model. Large errors should be avoided, but extreme precision is not required when slower but more exact methods are available for backup calculations.

What is described in the following section is a new technique of obtaining approximate solutions to variational trajectory problems. A fundamental point to make here is that this method does not involve approximate solutions to equations of motion for a variational trajectory problem. Instead, the principal idea is to develop general relations among the various "modes" of rocket operation based on the dynamics of their trajectory solutions. These modes of rocket operation may be impulsive (very high thrust), constant acceleration, constant thrust and jet velocity (power-coast-power), constant jet power with variable thrust, and others. In this way, any one solution of a given trajectory problem by a specific mode of rocket operation may be used as a reference. Energy requirements of the given trajectory problem for other operating modes are obtained by correlation with the reference mode solution, using the appropriate dynamic relations. Hence, a more descriptive term for this method is correlation.

The necessary dynamic relations will be based on analytic solutions of a simple problem. The result will be a "linking parameter" that is used in an analogous manner as the familiar ΔV of high-thrust trajectory analysis. This parameter can actually replace ΔV because it is a near-invariant factor within and among high- and low-thrust modes of operation.

Characteristic Velocity and Length Increments

The characteristic velocity increment ΔV is a familiar parameter of trajectory analysis. When used as an invariant of trajectory solutions in the case of high-thrust trajectory analysis, propellant requirements, and vehicle performance are generalized with respect to jet velocity. However, ΔV has not been as useful in low-thrust trajectory analysis because it is apparently a highly variable function of the thrust or mode of rocket operation.

In actuality, ΔV is the velocity increment that a rocket would experience on a rectilinear flight path in field-free space. This definition is directly traceable back to the so-called ideal rocket equation, which is the equation of motion in this system:

$$dV = a\ dt \tag{1a}$$

where V is velocity, a is acceleration due to thrust, and t is time. Alternatively,

$$dV = \frac{F}{m} dt \tag{1b}$$

where F is thrust and m is mass, and so

$$\Delta V = \int |a| dt \tag{2}$$

An equivalent ΔV can be evaluated for any trajectory solution by using appropriate expressions for the time integral of acceleration magnitude. For example, a familiar expression for ΔV is,

$$\Delta V = -v_J \ln \frac{M_f}{M_o} \tag{3}$$

where v_J is jet velocity. Equation (3) is simply a solution of equation (2) for constant thrust and jet velocity.

Every rectilinear trajectory also has a definable characteristic length increment L.

$$L = \int V\ dt \tag{4}$$

This concept of a characteristic length increment L is important in the ensuing development of simple dynamic relations among various modes of rocket operation.

It will be shown in the solution of simple rest-to-rest, rectilinear trajectories in field-free space, that basic energy requirements depend on the mode of rocket operation, travel time, and L. In this way, the dynamics of all modes of operation can be interrelated through L and travel time T.

Expressions of this type are easily developed for this simple problem. Similar forms have been used by other authors.[2,7] The contribution of this paper is in the further application of such expressions in a correlation-approximation method.

As with ΔV, an equivalent L can then be evaluated for any trajectory solution in the inverse-square force field. It will be shown that if this equivalent L is treated as an invariant of the trajectory problem, it becomes the linking parameter mentioned earlier.

A straightforward procedure for correlating trajectory energy requirements between any two modes of rocket operation can be easily constructed with relations between L and propulsive requirements.

The first step is a reference solution of the specific trajectory problem in the inverse-square force field by any favored mode of rocket operation. A point that must be emphasized here is that the reference-mode solution must pertain to the same trajectory problem that is of interest in the new mode. Travel time, travel angle, initial and final positions and velocities must agree. The second step is the evaluation of equivalent L from the appropriate propulsive energy requirement relation for the reference mode. The final procedure is the evaluation of propulsive requirements in the new mode based on the use of the characteristic length with the appropriate dynamic relation.

All that is required further are relations between L and propulsive requirements for each mode of rocket operation for rectilinear, field free, rest-to-rest, and trajectory solutions.

Rectilinear Trajectory Solutions

In figure 5, two simple examples of rectilinear, field-free, rest-to-rest trajectory solutions are developed. The first (a) is for infinite-thrust or impulsive-thrust solutions, while the second (b) is representative of low-thrust solutions. For simplicity, the low-thrust example is for constant acceleration without a coast period.

The infinite-thrust solution is shown graphically by its velocity chronology. The acceleration history is not shown since it only consists of two infinite impulses. The first impulse with infinite acceleration changes the velocity from zero to V_{max}, which is the value required to bring the rocket to position L at time T. The rocket proceeds at constant velocity V_{max} until time T, when it is brought back to rest instantaneously by applying a second impulse equal to V_{max}. Therefore, for the impulsive-thrust solution:

$$L = V_{max} T \tag{5}$$

and

$$\Delta V = 2V_{max} = \frac{2L}{T} \tag{6}$$

With constant acceleration, velocity varies linearly with time. Since it has been specified that there be zero velocity at the terminals and no coast phase, the velocity diagram must be an isosceles triangle. The velocity increases linearly from zero to V_{max} at half time $T/2$ and then must decrease linearly to zero again at time T. To accomplish this, the acceleration must be directed forward for half the trip and then reversed for the remaining half.

Since L is the area of the velocity triangle, it can be shown to be a function of the peak velocity V_{max} and T:

$$L = V_{max} \frac{T}{2} \tag{7}$$

V_{max} is the product of acceleration and $T/2$:

$$V_{max} = a \frac{T}{2} \tag{8}$$

Combining equations (7) and (8) shows the necessary acceleration to depend upon L and T:

$$a = \frac{4L}{T^2} \tag{9}$$

Previously, ΔV has been defined as the total integral of the acceleration magnitude. In this case, for constant acceleration then

$$\Delta V = \int a\ dt = aT = \frac{4L}{T} \tag{10}$$

If the result of equation (10) is compared with the corresponding result for the impulsive solution in equation (6), it is seen that, for these rectilinear trajectory problems, the ΔV for a constant-acceleration no-coasting solution is simply twice the ΔV of the impulsive-thrust solution of the same trajectory problem. This observation provided the fundamentals for the correlation solution of inverse-square force field trajectory problems, because this predicted relation holds nearly true for inverse-square trajectory solutions. Note the fact that ΔV has been shown to vary by as much as 100 percent, if it is assumed that the equivalent L of a trajectory problem is an invariant.

With little difficulty, equations that relate the equivalent characteristic length to propulsive energy requirements may be derived for other modes of rocket operation. A few are summarized here, without development:

Constant acceleration with coasting:

$$L = \frac{a}{4}\left(T^2 - t_c^2\right) \tag{11}$$

$$\Delta V = \frac{4L}{T + t_c} \tag{12}$$

where t_c is coast time. ΔV varies inversely with coast time. If coast time is zero, equation (12) reduces to equation (10). If coast time is made equal to T, equation (12) reduces to equation (6).

Constant thrust and constant jet velocity all-propulsion (no coast):

$$L = \frac{v_J^2}{a_o}\left(1 - \sqrt{1 - \frac{a_o T}{v_J}}\right)^2 \tag{13}$$

$$a_o = \frac{4L}{T^2}\frac{v_J^2}{\left(v_J + \frac{L}{T}\right)^2} \tag{14}$$

$$\Delta V = 2v_J \ln\left(\frac{v_J + \frac{L}{T}}{v_J - \frac{L}{T}}\right) \tag{15}$$

where a_o is initial acceleration, v_J is jet velocity, and T is transfer time. With constant thrust and jet velocity, mass flow rate is constant and therefore acceleration a must be treated as a variable. All-propulsion with constant thrust and jet velocity is simply a special case of the power-coast-power mode. For all-propulsion cases, the a_o is dependent on L, v_J, and T because the coast time is specified as zero. Equations (14) and (15) evaluate the minimum a_o and the maximum ΔV of constant-thrust trajectories when characteristic length is known.

Constant thrust and constant jet velocity power-coast-power (with coast):

$$L = \frac{v_J^2}{a_o}\left[\left(1 - \sqrt{1 - \frac{a_o t_p}{v_J}}\right)^2 - \frac{a_o(T - t_p)}{2v_J}\ln\left(1 - \frac{a_o t_p}{v_J}\right)\right] \tag{16}$$

$$\Delta V = \frac{2a_o L - 2v_J^2\left(1 - e^{-\Delta V/2v_J}\right)^2}{a_o T - v_J\left(1 - e^{-\Delta V/v_J}\right)} \tag{17}$$

where t_p is propulsion time. Equation (17) is derived from equation (16) and requires an iterative solution for ΔV when L, T, v_J, and a_o are given.

Constant jet power with variable thrust:

$$L = \sqrt{\frac{JT^3}{12}} \tag{18}$$

$$a_o = \frac{6L}{T^2} \tag{19}$$

where $J \equiv \int a^2\, dt$.

Equation (18) can serve to determine equivalent characteristic length of a trajectory problem based on variable thrust solutions in the inverse-square force field.

A discussion of variable thrust trajectories is beyond the scope of this paper. Equations (18) and (19) have nevertheless been included because variable-thrust trajectory solutions are a potentially good source of reference values of characteristic length, since they are more easily obtained than power-coast-power solutions.

Alternate parametric forms of equations (11) to (19) can be developed if desired.

Accuracy Comparisons

In figure 6, the approximation is compared with actual calculus of variations solutions for one-way heliocentric trajectories from Earth to Mars. With travel angle and time fixed, a curve of actual solutions is shown for power-coast-power trajectories with specific impulse fixed at 6000 seconds. This curve joins with another curve consisting of all-propulsion solutions along which specific impulse varies between 1000 seconds and infinity. The equivalent ΔV evaluated from the jet velocity and the actual mass ratio is shown for a wide range of initial acceleration from less than 10^{-3} meters per second squared to infinity. The approximate curves shown here faithfully follow the characteristics of the "exact" solutions. All the approximate ΔV's shown are calculated from equations (15) and (17). The equivalent L was obtained from the impulsive (infinite-thrust) solution of the same problem by using equation (6).

Since the length was evaluated from an impulsive-thrust solution, errors in ΔV are largest at the extremely low accelerations and are zero when acceleration is infinite.

The implementation of the correlation method presented is dependent on the assumed invariancy of equivalent characteristic length in actual inverse-square trajectory solutions. In the actual case, L is not truely invariant, and this is the source of errors in the correlation. Equivalent L, for a

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given trajectory problem, is known to vary slightly between different modes of operation. Also, L varies within a given mode of operation. Figure 7 illustrates the variation of characteristic length within a mode for power-coast-power trajectory solutions. The actual equivalent L, in meters, for a fixed trajectory problem is shown to vary with initial acceleration. Equation (16) has been used with the data from figure 2 to produce the actual equivalent length at each acceleration. Note that L varies by about 6 percent between the all-propulsion and infinite thrust cases. Most of the variation in L takes place at low accelerations (below 10^{-2} m/sec^2).

The most accurate evaluation of L would be obtained from a low-thrust solution that is close to the particular area of interest. The infinite-thrust solution, however, is currently the only available means of solving the boundary value problem with real speed and flexibility. It yields a usable value of characteristic length for approximations to the low-thrust solution at a speed far beyond any other solution method known. Therefore, for the most approximate but also the most useful application of this method to date, L is being evaluated from simple impulsive-thrust transfers with two-body conic sections.

The correlation method may be used with in infinite-thrust trajectory solutions to produce optimum probe and optimum round-trip performance charts such as illustrated in figures 3 and 4. The correlation is not in itself a variational solution, so trial-and-error methods are used to obtain optimum trajectory parameters. Nevertheless, the use of the correlation in this way provides a speed advantage of about 20 to 1 when compared with the time required to produce the variational data. Of course, errors in final mass ratio are presented in any mission performance chart generated with this use of the correlation method, since L is obtained from impulsive-thrust solutions. Errors in final mass are largest for all-propulsion solutions. A typical maximum error in final mass for Earth-Mars trajectories is about 5 percent of the initial mass.

CONCLUDING REMARKS

Trajectory calculations are shown to be a difficult, but necessarily integral, part of mission analysis for low-thrust vehicles. Although "exact" variational trajectory studies can be made, much time is consumed, which makes extensive application of these methods prohibitive.

A correlation approach for evaluating trajectory energy requirements has been described that satisfies the need for rapid, albeit approximate, trajectory analysis techniques. One application of this correlation method has shown a 20 to 1 speed advantage in performing trajectory analysis. A calculation speed advantage for the correlation approach is even further enhanced by being less complicated to use than numerically integrated variational trajectory methods. It is also more flexible in adaptation to various mission profiles of interest.

More importantly, the high-speed approximate trajectory solution readily lends itself to all-inclusive mission analysis techniques that go beyond mere trajectory analysis. The magnitude of the low-thrust trajectory problem has been markedly reduced. Hence, approximate trajectory calculation methods and realistic performance functions for vehicle subsystems are easily included in a mission analysis computer program wherein preliminary vehicles and mission profiles may be analyzed and optimized. Such a program is in use and has shown an effective speed advantage of 100 to 1 over mission analysis based on variational trajectory methods.

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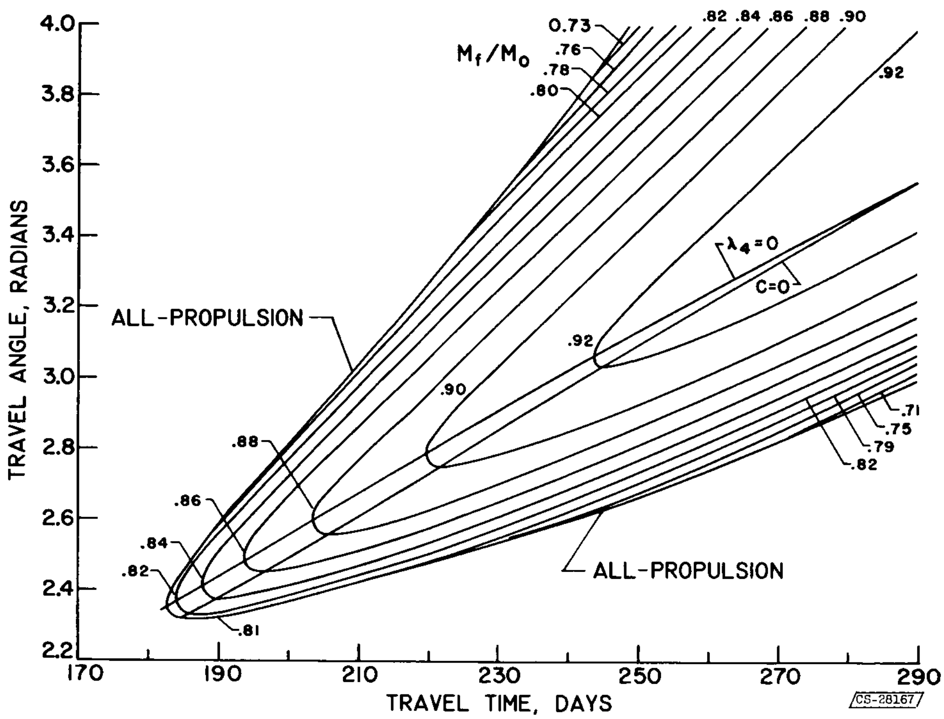

Fig. 1. - Effect of travel angle and travel time. Earth-Mars transfers $F/W_0 = 1 \times 10^{-4}$; I = 8,000 sec. Planetocentric spirals not included.

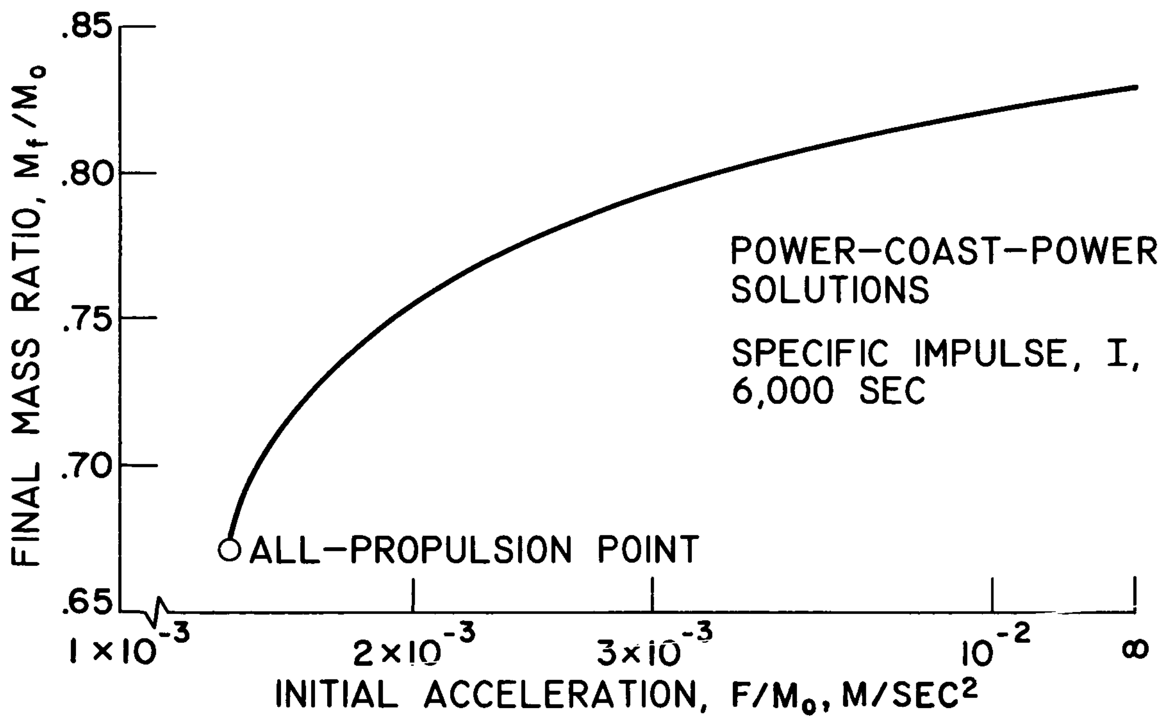

Fig. 2. - Variation of final mass with initial acceleration. One-way heliocentric trajectory to Mars. Travel time, 140 days; travel angle, 1.8 radians.

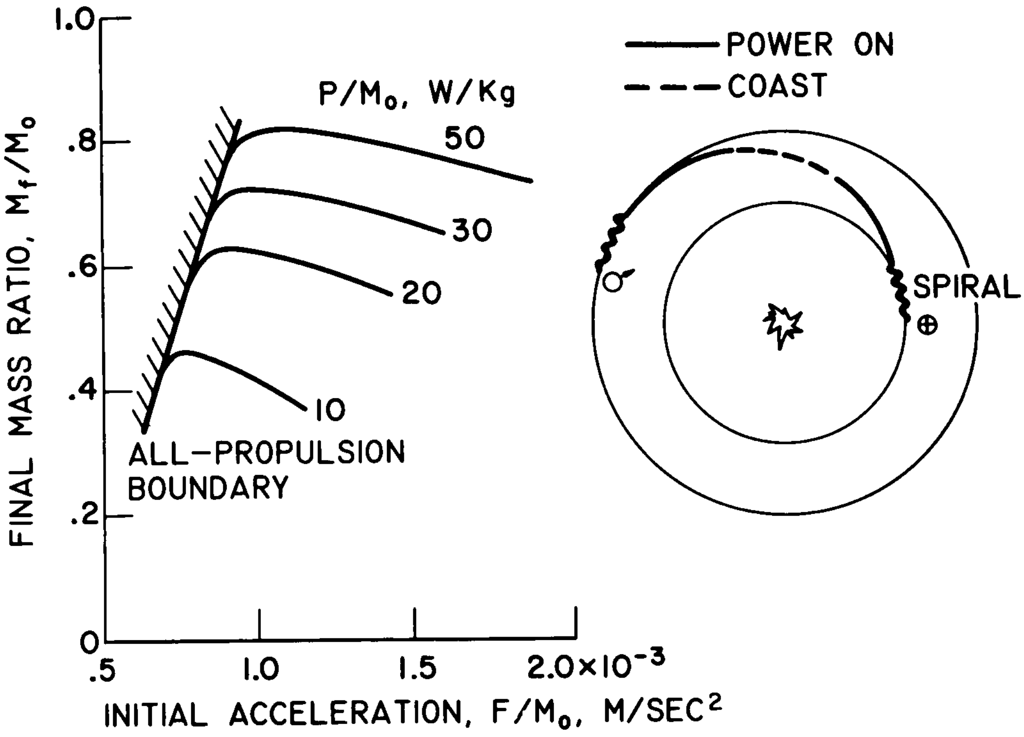

Fig. 3. - Typical chart of optimum orbiting probe solutions. Low Earth orbit to low Mars orbit; total travel time, 300 days.

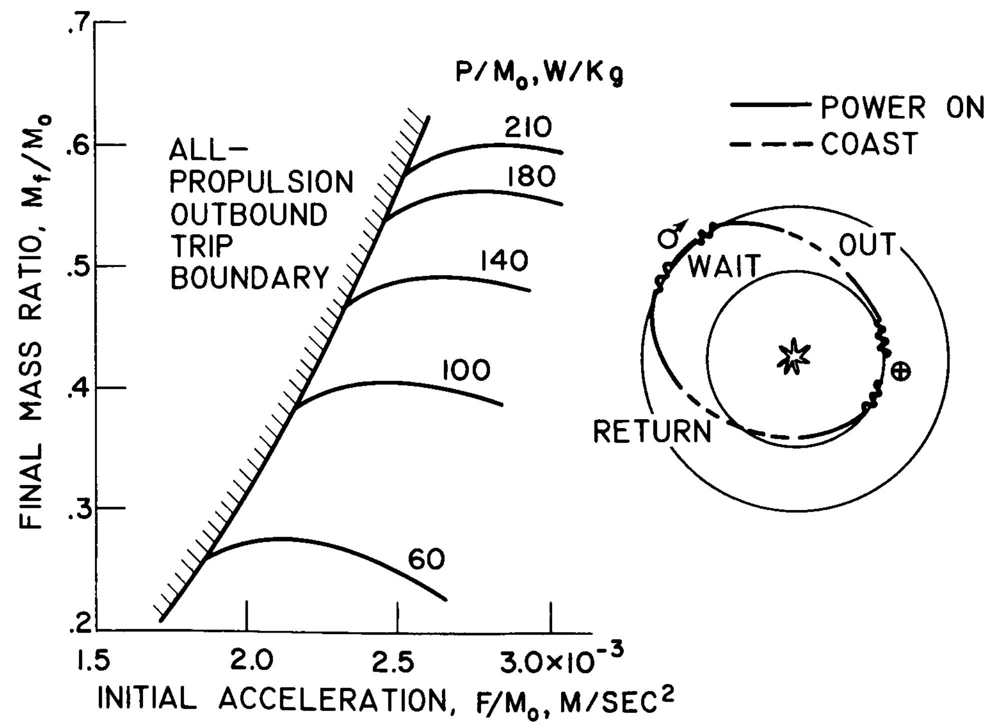

Fig. 4. - Typical chart of optimum round-trip solutions. Low Earth orbit to low Mars orbit and return. Mission time, 380 days; wait time, 10 days.

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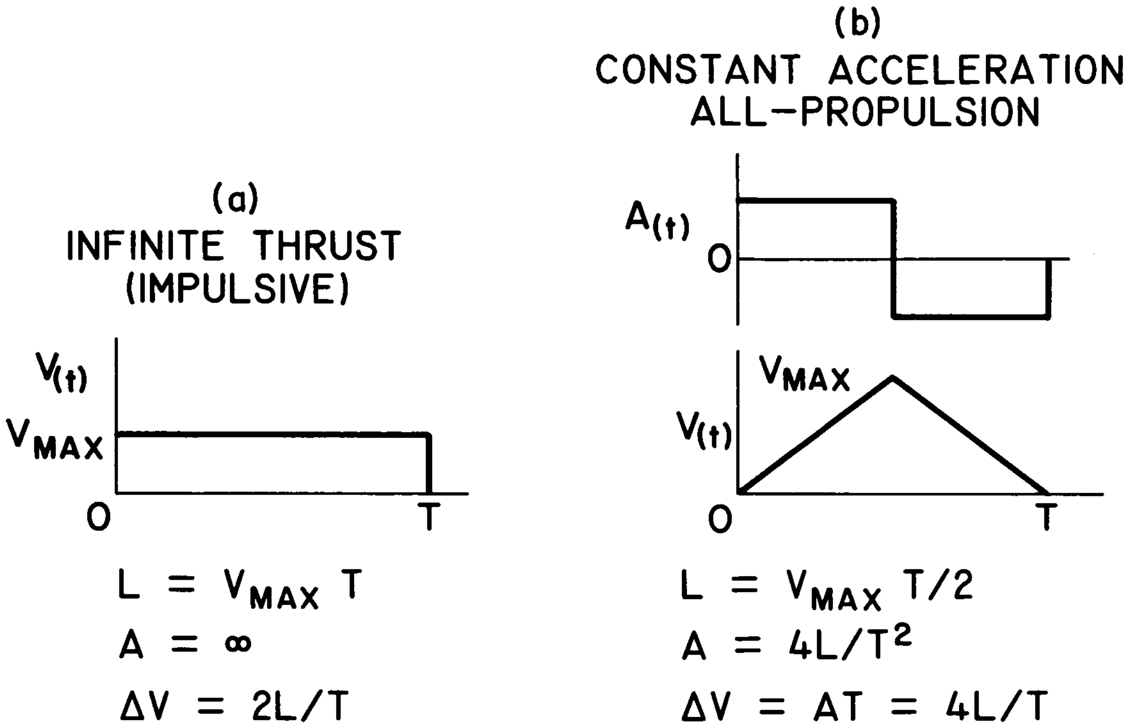

Fig. 5. - Solution of rectilinear trajectory problem. Rest-to-rest motion in field-free space.

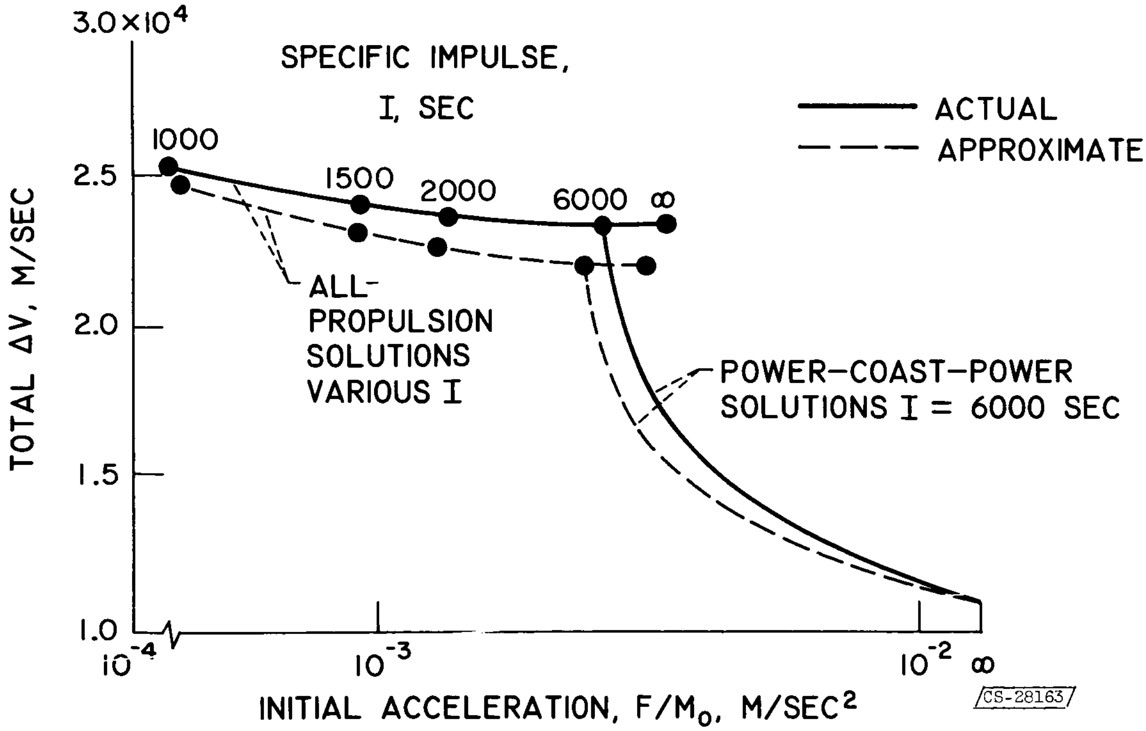

Fig. 6. - Comparison of variational and approximate solutions. One-way heliocentric trajectory to Mars. Travel time, 140 days; travel angle, 1.8 radians.

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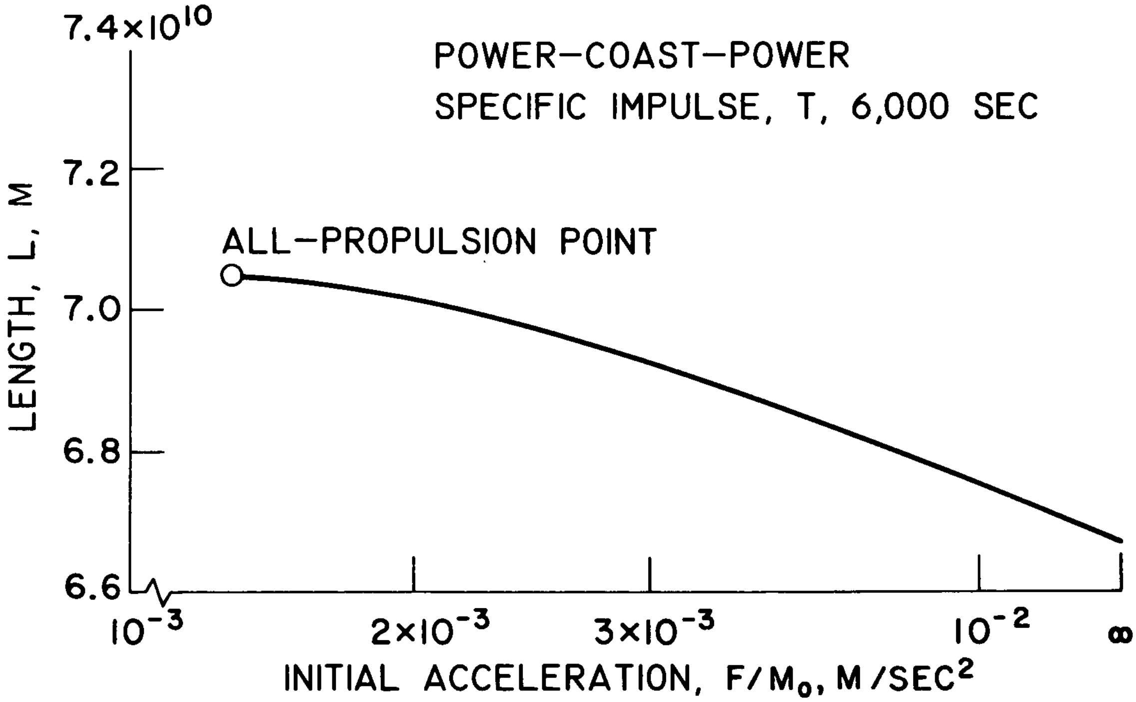

Fig. 7. - Variation of equivalent length. One-way heliocentric trajectory to Mars. Travel time, 140 days; travel angle, 1.8 radians.